

Anomalous QCD Contribution to the Debye Screening in an External Field via Holography.

A. Gorsky

ITEP, Moscow

P. N. Kopnin and A. Krikun

ITEP, Moscow and

MIPT, Moscow

(Dated: February 16, 2011)

Abstract

In this paper we discuss the QCD contribution to the Abelian Debye and magnetic screening masses in a deconfined QCD plasma at finite temperature in the presence of an external magnetic field B . We use a holographic AdS/QCD setup in an AdS Schwarzschild black hole background and show that the electric screening mass has a form similar to the one-loop result in QED. Moreover, we calculate the corrections due to the magnetic field to all orders of B and demonstrate that in the case when magnetic field is large the Debye mass grows linearly with B , while the magnetic screening mass vanishes. The whole effect of the magnetic field turns out to stem from the Chern–Simons action. We also discuss the zero temperature case in the chiral perturbation theory.

I. INTRODUCTION

Debye screening is a well-known effect in quantum field theory. In a hot plasma the static test charge is screened by real or virtual charged particles of the medium. The screening potential has a form $\frac{1}{r} e^{-r/l_D}$, where l_D is the Debye screening length. This potential emerges since photon acquires the effective nonzero Debye mass $m_D = l_D^{-1}$ in certain external conditions. A common method to calculate the screening mass is to study the infrared behavior of the polarization operator of the photon [1], [2]. By definition, the electric screening mass is

$$m_D^2 = e_q^2 \Pi_{00}(\omega = 0, \vec{k}^2 = -m_D^2), \quad (1)$$

where

$$\Pi_{\mu\nu}(\omega, \vec{k}) = \int d^4x \langle J_\mu(0) J_\nu(x) \rangle_{ret} e^{i\omega x_0 - i\vec{k}\vec{x}} \quad (2)$$

is a polarization operator of the photon corresponding to a retarded Green function and e_q is the charge with respect to the current J . A similar quantity related to the Π_{33} component of the polarization operator is called the magnetic screening mass and reflects the screening of the Lorentz force between two parallel currents:

$$m_{D \text{ Mag}}^2 = e_q^2 \Pi_{33}(\omega = 0, \vec{k}^2 = -m_D^2). \quad (3)$$

In QED the Debye mass at nonzero temperature is calculated in perturbation theory using the “hard thermal loop” approximation. The one loop result has been computed in [3] and equals

$$m_D^2 = \frac{e^2 T^2}{3}. \quad (4)$$

In [4] this quantity has been computed up to the e^5 order of perturbation theory. It is also interesting to study the dependence of the Debye mass on an external magnetic field in context of heavy ion collisions, where sufficiently large magnetic fields [5, 6] may exist. The behavior of the photon polarization operator and consequently the screening length in nonzero external field was studied in [7] by means of the Schwinger proper time formalism [8]. The magnetic screening mass can be shown to vanish to all orders of the perturbation theory [4].

The purpose of this paper is to compute the QCD effects on the screening of the electromagnetic interaction. Clearly, the quark loop enters the polarization operator of the photon, but

one can not limit oneself to one-loop approximation, because the coupling constant of QCD is not small (at least unless the temperature is too high). We study the QCD contributions to the screening masses in two cases. At temperatures higher than the temperature of the deconfinement phase transition, but low enough for the non-perturbative treatment of QCD ($1 \text{ GeV} \gtrsim T \gtrsim 200 \text{ MeV} \approx T_c \sim \Lambda_{QCD}$), we use the AdS/QCD model in the background of an AdS black hole (BH) [9–12]. To study the behavior of the Debye mass in an external magnetic field in the case with confinement we use the chiral perturbation theory approach at zero temperature [13] with the anomalous Wess–Zumino–Witten term.

In the holographic calculation we are able to treat the external magnetic field to all orders of the perturbation theory, thus obtaining an exact analytical result for any values of B . It turns out that the dependence on the external field is fully driven by the Chern–Simons interaction in the action of the AdS/QCD model. The Debye mass grows linearly with the magnetic field in the strong field limit, thus coinciding with the behavior found in [7] via the mode analysis. At zero magnetic field the non-perturbative calculation gives the value of the electric screening mass equal to the one-loop result in perturbation theory (4). We also confirm non-perturbatively the vanishing of the magnetic screening.

The paper is organized as follows. In Section II we give a brief introduction to the AdS/QCD methods and explore the deconfinement region of the QCD phase diagram without the external magnetic field. Section III is devoted to holographical calculations in the same phase with an external magnetic field. In Section IV we study the confinement regime via the chiral perturbation theory. The conclusion is given in Section V.

II. DECONFINED PHASE. $B = 0$

In this section we set about calculating the Debye screening mass m_D in the absence of the magnetic field as defined in Eq. (2), as well as the magnetic screening mass $m_{D \text{ Mag}}$ which is defined analogously in Eq. (3).

Since we are only interested in the QCD contributions to the mass we therefore ignore all contributions of order of $\alpha_{em} = e^2/4\pi$. One can easily notice that $\Pi_{\mu\nu}(\omega = 0, \vec{k}^2 = -m_D^2) - \Pi_{\mu\nu}(\omega = 0, \vec{k} = 0) \propto \alpha_{em}$, hence in what follows we shall study $\Pi_{\mu\nu}(\omega = 0, \vec{k} = 0)$.

In order to obtain the screening masses, according to (2, 3) one has to calculate a certain two-point function. A holographic prescription for this calculation [9] states that one has to identify five-dimensional fields dual to operators in question and assign them boundary values equal to the sources of these operators (hence the boundary conditions for the fields at the AdS boundary $r = \infty$ are fixed). A classical five-dimensional action is then identified with the logarithm of the quantum field theory generating functional. Therefore, to calculate a correlator via holography one has to vary the classical action in the AdS with respect to the boundary values of the relevant fields. In the calculation below we shall consider two types of correlation functions: the correlator of temporal components of the electromagnetic currents in the case of the Debye mass and the correlator of spatial components for the magnetic screening mass. Hence we shall introduce the sources to these currents in the corresponding cases. When investigating effects at zero momentum it is quite handy to introduce a chemical potential μ as a source of the temporal component of the vector current. The source for the spatial component will be denoted as j .

Let us consider the action of a holographic AdS/QCD model that yields a dual description of QCD:

$$S = S_{YM}[L] + S_{YM}[R]; \quad S_{YM}[A] = - \frac{2}{8g_5^2} \int F \wedge *F.$$

It is a standard Abelian gauge sector of the AdS/QCD action, and according to the AdS/CFT prescription the gauge fields are dual to the QCD currents under consideration. For the sake of simplicity, we are considering here a case with one quark flavor which corresponds to the Abelian action, but a generalization to N_f flavors is straightforward and does not lead to any qualitative changes in our results. Note that the action has an additional factor 2 as compared to that of the non-Abelian gauge fields [14]. This factor appears because the OPE of the two-point correlation function of the current $J_\mu = \bar{q}_f \gamma_\mu q_f$ of a particular quark flavor f , which couples to the photon and is dual to the 5D gauge field A_μ in our model, has the same factor 2 as compared to the OPE of flavor-nonsinglet currents $J_\mu^a = \sum_{f, f'} \bar{q}_f \gamma_\mu (t^a)_{ff'} q_{f'}$ [15]. The g_5 is a 5D coupling constant and is related to the number of colors $\frac{R}{g_5^2} = \frac{N_c}{12\pi^2}$ [14]. The action may

also be rewritten in terms of vector and axial gauge fields: $L = V + A$, $R = V - A$.

$$S = -\frac{1}{2g_5^2} \int dr d^4x \sqrt{-g} (F_{MN}^V F^{V\ MN} + F_{MN}^A F^{A\ MN}). \quad (5)$$

At temperatures under consideration the quark condensate and all mesons are melted, and thus the part of AdS/QCD action, responsible for a bifundamental scalar (cf. [14]), is absent.

The metric has the form:

$$ds^2 = \frac{r^2}{R^2} (-f_{BH}(r) dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} \frac{dr^2}{f_{BH}(r)}, \quad f_{BH}(r) = 1 - \frac{r_0^4}{r^4}. \quad (6)$$

of an AdS Schwarzschild black hole [10]. R is the AdS curvature radius, $r = \infty$ corresponds to the AdS boundary, and the BH radius r_0 is related to the temperature of the plasma:

$$T = \frac{r_0}{\pi R^2}. \quad (7)$$

Usually the presence of a nonzero chemical potential manifests itself as a charge of a Reissner–Nordström black hole, thus altering the expression for the metric in (6). However, in our calculations we are dealing only with two-point correlators at zero chemical potential. The main terms of the action itself are quadratic in μ , while the account of the black hole charge will yield the terms $\propto O(\mu^3)$, $\mu \rightarrow 0$, that cannot contribute to the two-point functions at zero chemical potential. Therefore in what follows we will keep the metric in the form of Eq. (6).

It has been pointed out in [11, 12] that calculations of retarded Green functions in AdS/CFT imply certain boundary conditions at the horizon $r = r_0$: we have to make sure that we leave only in-falling waves, which are solutions that are regular at the horizon in the corresponding Eddington–Finkelstein coordinates (see also [17]). One can easily see that in the case of a zero frequency this condition is equivalent to the regularity of solution in standard AdS coordinates:

$$A_i(r) \text{ and } V_i(r) \text{ are regular at } r = r_0, \quad (8)$$

where $i = 1, 2, 3$. In addition, temporal components of the gauge fields have to vanish at the horizon due to the fact that $g_{00}(r = r_0) = 0$:

$$A_0(r_0) = V_0(r_0) = 0. \quad (9)$$

As was pointed out earlier, the boundary condition at $r_0 \rightarrow \infty$ is determined by the source of the corresponding operator, namely

$$V_0(r = \infty) = \mu, \quad V_3(r = \infty) = j, \quad (10)$$

Let us start with the simplest case of the electric screening mass (2) in the absence of an external field. Without the magnetic field the vector and the axial gauge fields decouple. The only nonzero component of the vector field in the present case is V_0 , hence the action (5) is reduced to :

$$S = \frac{N_c}{12\pi^2 R^4} \int_{r_0}^{\infty} d^4x dr r^3 (\partial_r V_0(r))^2.$$

Equation of motion for V_0 is quite trivial:

$$\partial_r(r^3 \partial_r V_0) = 0.$$

The solution that takes into account the boundary condition at the BH horizon (9) and at the boundary (10) reads as :

$$V_0(r) = \mu \left(1 - \frac{r_0^2}{r^2} \right),$$

and determines the value of the on-shell action:

$$S = V_{4D} \frac{\mu^2 N_c r_0^2}{6\pi^2 R^4},$$

where V_{4D} is the 4D volume. According to the holographic prescription the correlator $\Pi_{00}(\omega = 0, \vec{k} = 0) = \frac{1}{V_{4D}} \frac{\partial^2 S}{\partial \mu^2}$ yields the following value of the Debye mass

$$m_D^2 = \frac{N_c}{3} e_q^2 T^2. \quad (11)$$

Interestingly enough, the result of a non-perturbative QCD calculation (11) is similar to the leading term of the QED perturbation series expression for the Debye mass (4) in [3].

Concerning the magnetic screening let us note that the equation of motion for the spatial components of the vector field V_i

$$\partial_r (r^3 f_{BH}(r) \partial_r V_i(r)) = 0 \quad (12)$$

allows only one solution which is regular at the horizon: $V_i(r) \equiv const = V_i(\infty)$. Thus the action for the spatial components $\propto \int dr r^3 f_{BH}(r) (\partial_r V_i(r))$ is zero, implying that the magnetic screening mass is zero:

$$m_{D \text{ Mag}} = 0. \quad (13)$$

This result is in agreement with a statement that $m_{D \text{ Mag}}$ is zero to all orders of the perturbation theory (see e.g. [4]).

III. DECONFINED PHASE. $B \neq 0$

A. The action

In this section we shall introduce the magnetic field by means of the Chern–Simons (CS) action, see [17, 18]. The full action of the model is now a sum of Yang–Mills (5) and Chern–Simons terms.

$$S = S_{YM}[L] + S_{YM}[R] + S_{CS}[L] - S_{CS}[R] \quad (14)$$

$$\begin{aligned} S_{CS}[A] &= -\frac{N_c}{24\pi^2} \int A \wedge F \wedge F - \frac{1}{2} A \wedge A \wedge A \wedge F + \frac{1}{10} A \wedge A \wedge A \wedge A \wedge A \\ &= -\frac{N_c}{24\pi^2} \int dz \, d^4x \, \epsilon^{MNPQR} A_M F_{NP} F_{QR}. \end{aligned} \quad (15)$$

In the Abelian case only the cubic term in the CS action is relevant. In terms of the vector and axial fields $L = V + A$, $R = V - A$ it assumes the form:

$$\begin{aligned} S_{CS} &= \frac{-N_c}{4\pi^2} \int dr \, d^4x \, \epsilon^{MNPQR} A_M F_{NP}^V F_{QR}^V + \frac{-N_c}{12\pi^2} \int dr \, d^4x \, \epsilon^{MNPQR} A_M F_{NP}^A F_{QR}^A \\ &\quad + \frac{-N_c}{6\pi^2} \int d^4x \, \epsilon^{\mu\nu\lambda\rho} A_\mu V_\nu F_{\lambda\rho}^V \Big|_{r=r_0}^{r=\infty}. \end{aligned} \quad (16)$$

The Chern–Simons term gives rise to the interaction with the external magnetic field. and $F_{12}^V(r = \infty)$ is associated with the magnetic field multiplied by the electric charge, $e_q B$.

There are two ways to obtain the expressions for the screening masses in this setting. The first one is to treat the problem perturbatively, considering Feynman diagrams that contain various numbers of legs corresponding to the external magnetic field, which is carried out in Subsection III B. This consideration in its turn motivates a non-perturbative diagonalization of the action in the external field, which is performed in Subsection III C.

B. Diagrams

From the action (14) one gets equations of motion for spatial and temporal components of the vector field in the infrared limit ($\omega = 0, q^2 = 0$):

$$\begin{aligned} -\partial_r(r^3 \partial_r V_0(r)) &= (AV) \text{ interactions;} \\ -\partial_r(r^3 f_{BH}(r) \partial_r V_i(r)) &= (AV) \text{ interactions.} \end{aligned}$$

The bulk-to-boundary propagators v_0 and v_i are solutions to these equations without the interaction terms, subject to boundary conditions (9),(8), (10). (A more detailed study of the perturbation theory in question may be found in [19].) Two branches of the solution to the spatial equation are $v_i = 1$ (see Eq. (12)) and $v_i = \log\left(\frac{r^2-r_0^2}{r^2+r_0^2}\right)$. The latter diverges at the horizon ($r = r_0$), so we must omit it. Hence we end up with a trivial spatial bulk-to-boundary propagator:

$$v_i(r)|_{q,\omega=0} = 1. \quad (17)$$

If we consider the Chern–Simons action in Eq. (16), we observe that both the terms involving three axial fields and the term with one axial and two dynamical vector fields can contribute to the two-point correlators. Moreover we need to take into account only the first term with two dynamical axial and vectors fields and a vector field which stems from the external magnetic field propagating into the bulk. We also note that even in the case of a non-Abelian action (involving N_f quark flavors) vertices from the non-Abelian part of the Yang-Mills action do not contribute to the correlator under consideration (see [19]). We can depict the $\langle J_0 J_0 \rangle$ and $\langle J_3 J_3 \rangle$ correlation functions that determine the electric and the magnetic screening masses as a sum of diagrams (Fig. 1) that include the aforementioned vertex from the Chern–Simons term (while the correlators $\langle J_1 J_1 \rangle = \langle J_2 J_2 \rangle$ are discussed further below). From the action (16) we read out the vertex functional [19]:

$$\mathbb{A}_{\alpha\beta\gamma} = \delta^4(q_1+q_2+q_3) \epsilon^{\alpha\beta\gamma\sigma} \frac{r^2}{R^2} (\partial_r^2 q_\sigma^1 - \partial_r^1 q_\sigma^2). \quad (18)$$

Due to the epsilon symbol all interacting fields must have different Lorentz indices, hence there can only be one temporal component at each vertex. As the spatial bulk-to-boundary propagator is trivial, it can not be acted on by ∂_r , so the only way to get a nonzero result is to act by ∂_r on the temporal component. Furthermore, the momenta of all incoming photons are zero, hence in a tree-level diagram all momenta should be zero. Therefore the only way to place a spatial derivative is to act with it on the external field and obtain the dual field strength tensor $\tilde{F}^{\alpha\beta} = \epsilon^{\alpha\beta\gamma\sigma} q_\sigma V_\gamma$. Ultimately we find that the triple vertex boils down to a mixing term between the spatial component of the axial field and the temporal component of the vector field (Fig. 1a), or vice versa (Fig. 1b). If we choose the external magnetic field to

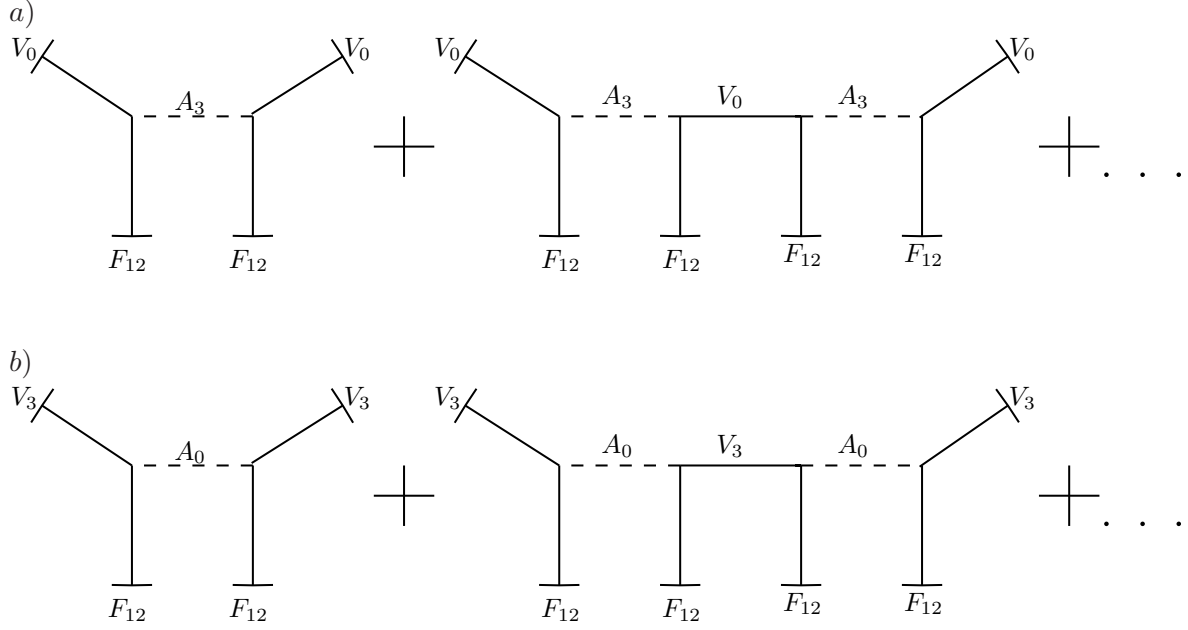


FIG. 1. Tree-level diagrams, corresponding to calculations of the vector current correlator in the external field F_{12} : a) temporal components $\langle J_0, J_0 \rangle$, b) spatial components $\langle J_3, J_3 \rangle$.

be $(0, 0, B)$, we get an effective interaction term :

$$\begin{aligned}
S_{CS} = & -\frac{N_c e_q B}{2\pi^2} \int d^4 x dr A_3(r) \partial_r V_0(r) + \frac{N_c e_q B}{6\pi^2} \int d^4 x A_3 V_0 \Big|_{r=r_0}^{r=\infty} \\
& + \frac{N_c e_q B}{2\pi^2} \int d^4 x dr V_3(r) \partial_r A_0(r) - \frac{N_c e_q B}{6\pi^2} \int d^4 x V_3 A_0 \Big|_{r=r_0}^{r=\infty}, \quad (19)
\end{aligned}$$

where the first line is relevant for the $\langle J_0 J_0 \rangle$ correlator — see Subsection III C 1 while the second one — the $\langle J_3 J_3 \rangle$ correlator — see Subsection III C 2.

A simple consideration of the diagrams demonstrates that the quantities Π_{11} and Π_{22} are identically zero due to the epsilon symbol in the vertex (18). Note however, that these quantities vanish only in the leading order of large N_c expansion. To find the $\frac{1}{N_c}$ corrections to this result, one should consider the diagrams with dilaton and graviton exchange in the bulk, which can produce nonzero input to Π_{11} and Π_{22} (see [19]). The consistent treatment of $\frac{1}{N_c}$ corrections involves taking account of higher orders of string perturbation theory, which is out of the scope of this paper. Nevertheless, we can state the result

$$\Pi_{11}, \Pi_{22} = O(1), \quad (20)$$

while $\Pi_{00} = O(N_c)$.

C. Diagonalization

1. Electric screening mass

Let us consider the Chern–Simons action in Eq. (16). As it was pointed out in the previous Subsection (IIIB), the contribution of the Chern–Simons action to the equations of motion is reduced to a mixing between the axial and the vector fields, see Eq. (19). The relevant part of the whole action involving the V_0 and A_3 fields assumes the form:

$$S = \frac{N_c}{12\pi^2 R^4} \int d^4x dr \left[r^3 (\partial_r V_0(r))^2 - r^3 f_{BH}(r) (\partial_r A_3(r))^2 - 6e_q B R^4 A_3(r) \partial_r V_0(r) + 2e_q B R^4 \partial_r (A_3(r) V_0(r)) \right]. \quad (21)$$

The corresponding equations of motion are:

$$-\partial_r(r^3 \partial_r V_0(r)) + 3e_q B R^4 \partial_r A_3(r) = 0; \quad (22)$$

$$\partial_r(r^3 f_{BH}(r) \partial_r A_3(r)) - 3e_q B R^4 \partial_r V_0(r) = 0. \quad (23)$$

As for the boundary conditions (see Section II), the values of the gauge fields at the AdS boundary ($r = \infty$) are determined by the sources (10), where we put $j = 0$ and keep only a source for V_0 : $V_0(\infty) = \mu$, $A_3(\infty) = 0$. Boundary conditions at the black hole horizon are determined by Eqs. (8, 9).

A general solution to Eqs. (22, 23) is:

$$V_0(r) = \int_{\infty}^r dr' (C_1 P_{\nu}(r_0^2/r'^2) + C_2 Q_{\nu}(r_0^2/r'^2)) + C_3, \quad (24)$$

$$A_3(r) = \frac{C_1}{\beta} P_{\nu}(r_0^2/r^2) + \frac{C_2}{\beta} Q_{\nu}(r_0^2/r^2) + C_4, \quad (25)$$

where $\beta = 3e_q B R^4$, $\nu = -\frac{1 - \sqrt{1 - \beta^2/r_0^4}}{2}$, $P_{\nu}(z)$ and $Q_{\nu}(z)$ are the Legendre functions of the first and second order respectively that are single-valued and regular for $|z| < 1$. In our case ν is real, varies from 0 ($B = 0$) to $-1/2$ ($e_q B = \pi^2 T^2/6$) and acquires an imaginary part for greater values of the magnetic field. Let us note that in the case of $x \in \mathbb{R}$ and $x \rightarrow 1$ $P_{\nu}(x)$ has

a finite limit, while $Q_\nu(x)$ possesses a logarithmic singularity (which corresponds to a branching point in the complex plane).

As one can see, the argument x of the Legendre functions in Eq. (24) varies from 0 to 1, the former corresponding to the AdS boundary and the latter – to the BH horizon. Therefore in order to have an axial field, regular at the horizon and zero at the boundary, according to Eqs. (8, 9, 24, 25) we should leave only the Legendre function of the first order in the solutions (24, 25):

$$C_2 = 0. \quad (26)$$

Values of the coefficients C_3, C_4 in (24, 25) are determined by the boundary conditions at the AdS boundary:

$$C_3 = \mu, \quad C_4 = \frac{C_1}{\beta} P_\nu(0), \quad (27)$$

while the coefficient C_1 is to be determined by the boundary conditions at the horizon (8, 9):

$$0 = V_0(r_0) = \mu - \frac{C_1}{2r_0^2} P_\nu^{-1}(0) \Rightarrow C_1 = \mu \frac{2r_0^2}{P_\nu^{-1}(0)}. \quad (28)$$

Thus, combining Eqs. (24, 25, 26, 27, 28), we find that the gauge fields have the following dependence on the radial coordinate r :

$$V_0(r) = \frac{\mu}{\nu P_\nu^{-1}(0)} \left(\frac{r_0^2}{r^2} P_\nu(r_0^2/r^2) - P_{\nu+1}(r_0^2/r^2) \right), \quad (29)$$

$$A_3(r) = \frac{\mu}{P_\nu^{-1}(0) \sqrt{-\nu(\nu+1)}} (P_\nu(r_0^2/r^2) - P_\nu(0)). \quad (30)$$

Taking into account the equations of motion – Eqs. (22, 23) – we get from Eq. (21):

$$S = \frac{N_c}{12\pi^2 R^4} \int d^4x \left[r^3 V_0(r) \partial_r V_0(r) - f_{BH}(r) r^3 A_3(r) \partial_r A_3(r) - \frac{\beta}{3} A_3(r) V_0(r) \right] \Big|_{r=r_0}^{r=\infty}. \quad (31)$$

According to the boundary conditions at the AdS boundary and at the BH horizon (8, 9) the second and the third in Eq. (31) do not contribute at all, while the first one is nonzero only at the boundary $r = \infty$. Hence,

$$S = \frac{N_c}{12\pi^2 R^4} \lim_{r \rightarrow \infty} (r^3 V_0(r) \partial_r V_0(r)) = \frac{N_c}{12\pi^2 R^4} 2r_0^2 \mu^2 \frac{P_\nu(0)}{P_\nu^{-1}(0)}.$$

Denoting the factor

$$F(\nu) \equiv \frac{P_\nu(0)}{P_\nu^{-1}(0)} = \frac{2\Gamma(1-\nu/2)\Gamma(3/2+\nu/2)}{\Gamma(1+\nu/2)\Gamma(1/2-\nu/2)} \quad (32)$$

we obtain the following exact analytical expression for the Debye mass in any external magnetic field:

$$m_D^2 = e_q^2 \frac{N_c}{3} T^2 F \left(-\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{9e_q^2 B^2}{\pi^4 T^4}} \right). \quad (33)$$

It is instructive to calculate this quantity numerically in the case of a quark-gluon plasma that is created during heavy-ion collisions at RHIC and at the LHC. If we use numerical values $T \approx 2T_c = 330 \pm 20$ MeV [21] and $|eB| \approx m_\pi^2 \approx 2 \times 10^4$ MeV² [5] for RHIC and $T \approx 4 - 5 \cdot T_c = 750 \pm 120$ MeV [22] and $|eB| \approx 15m_\pi^2 \approx 3 \times 10^5$ MeV² [6] for the LHC, we obtain

$$\begin{aligned} m_D^2 &= (82 \pm 3)^2 \text{ MeV}^2 \text{ at RHIC and} \\ m_D^2 &= (185 \pm 35)^2 \text{ MeV}^2 \text{ at the LHC.} \end{aligned} \quad (34)$$

In the case of a **weak magnetic field** $e_q B \ll T^2$ it is useful to expand the function $F(\nu)$ (32) in series of ν and take into account that $\nu = -\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{9e_q^2 B^2}{\pi^4 T^4}}$:

$$F \left(-\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{9e_q^2 B^2}{\pi^4 T^4}} \right) \approx 1 + 0.008923 \frac{e_q^2 B^2}{T^4} - 0.000021 \frac{e_q^4 B^4}{T^8} + O \left(\frac{e_q^6 B^6}{T^{12}} \right). \quad (35)$$

In the case of a **strong magnetic field** we can use the asymptotic behavior of the gamma function $\Gamma(z) \sim \sqrt{2\pi} e^{-z} z^{z-1/2}$, $|z| \rightarrow \infty$, and get

$$F \left(-\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{9e_q^2 B^2}{\pi^4 T^4}} \right) \sim \frac{3}{2\pi^2} \frac{|e_q B|}{T^2} \left(1 + \frac{17\pi^4}{216} \frac{T^4}{e_q^2 B^2} + O \left(\frac{T^8}{e_q^4 B^4} \right) \right). \quad (36)$$

Thus in the limit $e_q B \gg T^2$ the Debye mass turns out to be linear in the magnetic field, in a nice agreement with a weak coupling result in QED (see [7]):

$$m_D^2 = e_q^2 \frac{N_c}{2\pi^2} |e_q B|. \quad (37)$$

The dependence of the mass on the magnetic field is plotted on Fig. 2. The similarity of the dynamics of strongly coupled QCD and weakly coupled QED in large external magnetic fields is a nontrivial phenomenon, which was observed also in [20].

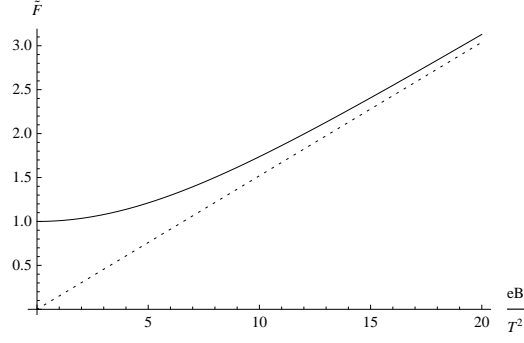


FIG. 2. The function $\tilde{F} = F \left(-\frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{9e_q^2 B^2}{\pi^4 T^4}} \right)$ (solid) vs its strong field asymptotics (dashed).

2. Magnetic screening mass

The calculation of the magnetic screening mass (3) is quite similar to one in the previous subsection. The only dynamical fields we need to consider are $V_3(0)$ and $A_0(r)$, the former having a source j with respect to which we have to variate the action twice in order to get the two-point correlator $\langle J_3 J_3 \rangle$, and the mixing due to the presence of the CS action proportional to the external magnetic field.

The action we are dealing with in this subsection is the following (see Eq. (19)):

$$S = \frac{N_c}{12\pi^2 R^4} \int d^4 x dr \left[r^3 (\partial_r A_0(r))^2 - r^3 f_{BH}(r) (\partial_r V_3(r))^2 + 6e_q B R^4 V_3(r) \partial_r A_0(r) - 2e_q B R^4 \partial_r (V_3(r) A_0(r)) \right]. \quad (38)$$

It generates the equations of motion analogous to Eqs. (23, 22):

$$-\partial_r(r^3 \partial_r A_0(r)) + 3e_q B R^4 \partial_r V_3(r) = 0; \quad (39)$$

$$\partial_r(r^3 f_{BH}(r) \partial_r V_3(r)) - 3e_q B R^4 \partial_r A_0(r) = 0. \quad (40)$$

The boundary conditions in this case are determined by the fact that V_3 has a source j at the AdS boundary while A_0 has none, and at the horizon — by Eqs. (8, 9). The only solution to Eqs. (40, 39) with these boundary conditions is:

$$A_0(r) \equiv 0, \quad V_3(r) \equiv j. \quad (41)$$

The corresponding action (38) is zero, therefore the magnetic screening mass is zero:

$$m_{D \text{ Mag}} = 0 \quad (42)$$

even in the presence of the magnetic field, while as already mentioned in [4] it is shown to vanish to all orders of perturbation theory in the absence of an external field. Our result is obtained in the leading order of $\frac{1}{N_c}$ expansion and, as in (20), can acquire subleading corrections. Nevertheless, let us note, that in QED ($N_c = 1$) a similar feature takes place: $m_{D \text{ Mag}} = 0$ in an external magnetic field [7].

D. Lower temperature case in holography

As we move to lower temperatures, expression in Eq. (33) asymptotically tends to Eq. (37) and thus the Debye mass grows linearly with the magnetic field. However, this result holds only if we impose the same boundary conditions in the infrared region of the AdS as in the case of high temperatures. In reality, the geometry of this region may change drastically when temperature T approaches the Λ_{QCD} scale, undergoing the Hawking–Page transition associated with the deconfinement phase transition of QCD [10]. One of AdS/QCD models [14] suggests, for instance, that in the confinement phase one places a hard wall at a certain point $r = r_m$ and impose a Neumann boundary condition for all fields at $r = r_m$, where $r_m \propto \Lambda_{QCD}^{-1}$:

$$\partial_r V_\mu(r_m) = \partial_r A_\nu(r_m) = 0, \quad (43)$$

where $\mu, \nu = 0, 1, 2, 3$. This hard wall is situated inside the BH at high temperatures and is uncovered by the BH horizon when the temperature drops lower than $\approx T_c \sim \Lambda_{QCD}$. If we consider the equations of motion (23, 22) with the new Neumann boundary conditions (43), a simple analysis demonstrates that their only possible solution is a pair of identically zero functions: $A_3(r) = V_0(r) \equiv 0$. Thus the Debye mass at temperatures below the phase transition appears to be zero in this particular model. The same holds true for the magnetic screening mass.

However, this result strongly depends on the type of the boundary conditions that we impose at the infrared boundary and may only be considered as a qualitative indication. We shall treat the case of a zero temperature more rigorously in the next section.

IV. CONFINEMENT PHASE

To study the screening masses in the confinement phase we make use of the Chiral Perturbation Theory (ChPT) [13]. As was shown in the previous sections, in the holographic approach the whole effect is governed by a Chern–Simons type interaction. Interestingly enough, in the chiral perturbation theory there exists a quite similar diagram, describing the correlation function of two vector currents in an external field (Fig. 3). It includes two anomalous vertices and an exchange of a π^0 meson.

Let us emphasize that the anomalous vertices correspond to the Goldstone–Wilczek currents emerging upon the computation of the fermionic loop in the varying meson field. That is, evaluating the Debye screening we actually look at the correlation of two induced electric charges if simultaneously magnetic and π_0 meson fields are switched on. To some extent this is a kind of a contribution which is quadratic in the effective chiral chemical potential.

As we are interested mainly in the effects of an external magnetic field, we restrict ourselves to the case of zero temperature. The relevant terms of ChPT Lagrangian in the external field are [13]:

$$L_{\chi PT} = \frac{1}{2} \partial_\mu \phi^\dagger \partial^\mu \phi + \frac{1}{2} M_\pi^2 \phi^\dagger \phi - \frac{\alpha_{em}}{4\pi} \frac{1}{f_\pi} \phi F_{\mu\nu} \tilde{F}_{\mu\nu} \quad (44)$$

As was mentioned above, the correction to the photon polarization operator in the external field arises already at the tree level and can be computed quite easily.

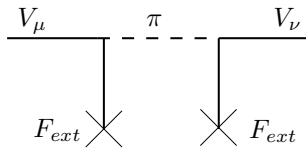


FIG. 3. The correction to the photon polarization operator in the external field in ChPT

It reflects the well-known photon-pion mixing in the magnetic field.

In the case when $B_{ext} = (0, 0, B)$ one gets the following result for $\Pi_{\mu\nu}(q, \omega)$ from the diagram

(Fig. 3):

$$\begin{aligned}\Pi_{00}(q, \omega) &= \frac{\alpha_{em}^2}{(4\pi)^2} \frac{B^2}{f_\pi^2} \frac{-q_3 q_3}{\omega^2 - |\vec{q}|^2 - M_\pi^2} \\ \Pi_{33}(q, \omega) &= \frac{\alpha_{em}^2}{(4\pi)^2} \frac{B^2}{f_\pi^2} \frac{-\omega^2}{\omega^2 - |\vec{q}|^2 - M_\pi^2}\end{aligned}\tag{45}$$

To obtain the screening masses (2),(3), we need to set $\omega = 0$ and then take $\vec{q}^2 = -m_D^2$. Then the magnetic mass associated with Π_{33} vanishes, coinciding with the result of the holographic calculation. However the Debye mass behaves much more interestingly. Suppose we consider the correlator with finite but small momentum $|\vec{q}|^2 = -m_D^2$. The 3rd component of the momentum can be expressed as $q_3 = m_D \cos(\widehat{\vec{q}, \vec{B}})$. Then the equation (45) yields:

$$m_D^2 \left(m_D^2 - M_\pi^2 - \frac{\alpha_{em}^2}{(4\pi)^2} \frac{B^2}{f_\pi^2} \cos^2(\widehat{\vec{q}, \vec{B}}) \right) = 0.$$

Naively this equation has two solutions, $m_D = 0$ and $m_D^2 = M_\pi^2 + \frac{\alpha_{em}^2}{(4\pi)^2} \frac{B^2}{f_\pi^2} \cos^2(\widehat{\vec{q}, \vec{B}})$. The second one arises due to the fact that the definition of the Debye screening in the confining phase in the magnetic field needs for some care. In the presence of a magnetic field in the confinement phase the photon mixes with the pion due to the anomaly. Therefore the Debye mass is naturally defined upon the diagonalization of two mixing states; one has to look for the poles of the propagator of the states. One of the poles $m_D^2 = 0$ corresponds to the Debye mass of the photon, while another reflects the shift of the pion mass due to an admixture of the photon, and therefore this pole is irrelevant here. Hence we get a vanishing anomalous contribution at zero temperature which seems quite natural.

Nevertheless let us point out that in the case when the pion is massless Eq. (45) has only one solution $m_D^2 = \frac{\alpha_{em}^2}{(4\pi)^2} \frac{B^2}{f_\pi^2} \cos^2(\widehat{\vec{q}, \vec{B}})$ which indicates the existence of an anisotropic deformation of the Coulomb potential at zero temperature in an external magnetic field. However let us stress that we have discussed in this Section the anomalous contribution only, while there is an additional contribution to the polarization operator of a loop with charged pions.

V. CONCLUSION

In this work we have studied the yet unexplored corrections to electromagnetic screening masses in a deconfined QCD plasma due to strong interactions. At temperatures, larger than

the temperature of deconfinement we have used the holographic AdS/QCD model to describe the QCD dynamics. The advantage of the holographic results we have obtained (33, 42) is the exact treatment of the external magnetic field, namely, all orders of perturbation theory have been summed up. Having an analytical formula for the Debye screening mass (33), we have studied various limits in the external field. Given that the external field is small we have found that the non-perturbative result (11, 35) equals that of the first order of the perturbation theory in QED (4) [3]. The behavior of the electric mass in a large magnetic field (37) coincides with a result of a calculation in one-loop QED in an external field and at nonzero temperature [7]. We have also found the magnetic screening mass to be zero at any values of the magnetic field.

It turned out that in our holographic model the dependence of the screening mass on the magnetic field is fully driven by the Chern–Simons term. Motivated by this fact, we have studied a similar diagram in the chiral perturbation theory and found an interesting anisotropy of the Debye mass in the magnetic field.

The obtained results show a nice agreement with all previous studies of the Debye screening and demonstrate the sensibility of the holographic model considered in this paper. It would be very natural to extend our Debye mass consideration to a dense QCD. In the deconfined phase such setup is holographically described by a charged black hole. On the other hand in the confining phase there are arguments that in a large magnetic field matter behaves as a stack of pionic domain walls [24]. Such an unusual state is stable both perturbatively [24] and non-perturbatively [25]. It would be very interesting to investigate the screening behavior in this phase as well.

ACKNOWLEDGMENTS

We would like to thank V. I. Shevchenko for the question initiated this research and A. V. Zayakin for fruitful discussions. Research of P. N. K. was supported by the Dynasty Foundation, the grant RFBR-09-02-00308 and by the Ministry of Education and Science of the Russian Federation under contract 14.740.11.0081. Research of A. K. was supported by the Dynasty Foundation, the grant RFBR-10-02-01483 and by the Ministry of Education and Science of

the Russian Federation under contract 14.740.11.0347. The work of A. G. is supported in part by the grants PICS- 07-0292165, RFBR-09-02-00308 and CRDF - RUP2-2961-MO-09. A. G. thanks IPhT at Saclay where the part of this work has been done for the hospitality and support.

-
- [1] J. I. Kapusta and C. Gale, “*Finite-temperature field theory: Principles and applications*,” Cambridge, UK: Univ. Pr. (2006) 428 p.
 - [2] M. Le Bellac, “*Thermal Field Theory*”, Cambridge Monographs on Mathematical Physics (1996) 256 p.
 - [3] H. A. Weldon, “Covariant Calculations At Finite Temperature: The Relativistic Plasma,” *Phys. Rev. D* **26**, 1394 (1982).
 - [4] J. P. Blaizot, E. Iancu and R. R. Parwani, “On The Screening Of Static Electromagnetic Fields In Hot QED Plasmas,” *Phys. Rev. D* **52**, 2543 (1995) ePrint arXiv: hep-ph/9504408.
 - [5] Dmitri E. Kharzeev, Larry D. McLerran, Harmen J. Warringa, “The Effects of topological charge change in heavy ion collisions: ‘Event by event P and CP violation’ ”. *Nucl. Phys. A* **803**, 227-253, 2008; e-Print arXiv: 0711.0950 [hep-ph].
 - [6] V. Skokov, A.Yu. Illarionov, V. Toneev, “Estimate of the magnetic field strength in heavy-ion collisions”, *Int. J. Mod. Phys. A* **24**, 5925-5932, 2009; e-Print arXiv: 0907.1396 [nucl-th].
 - [7] J. Alexandre, “Vacuum polarization in thermal QED with an external magnetic field,” *Phys. Rev. D* **63**, 073010 (2001) e-Print arXiv: hep-th/0009204.
 - [8] J. S. Schwinger, “On gauge invariance and vacuum polarization,” *Phys. Rev.* **82**, 664 (1951).
 - [9] J. M. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231-252 (1998), e-Print arXiv: hep-th/9711200,
S. S. Gubser, I. R. Klebanov and A. M. Polyakov, *Phys. Lett. B* **428**, 105 (1998), e-Print arXiv: hep-th/9802109,
E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998), e-Print arXiv: hep-th/9802150.
 - [10] E. Witten, “Anti-de Sitter space, thermal phase transition, and confinement in gauge theories,” *Adv. Theor. Math. Phys.* **2**, 505 (1998) e-Print arXiv: hep-th/9803131.

- [11] D. T. Son and A. O. Starinets, “Minkowski-space correlators in AdS/CFT correspondence: Recipe and applications,” *JHEP* **0209**, 042 (2002) e-Print arXiv: hep-th/0205051.
- [12] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, “Building a Holographic Superconductor,” *Phys. Rev. Lett.* **101**, 031601 (2008) e-Print arXiv: 0803.3295 [hep-th].
- [13] J. Gasser and H. Leutwyler, “Chiral Perturbation Theory To One Loop,” *Annals Phys.* **158**, 142 (1984).
- [14] Joshua Erlich, Emanuel Katz, Dam T. Son, Mikhail A. Stephanov, “QCD and a holographic model of hadrons”, SLAC-PUB-10965, WM-05-101, INT-PUB-05-02, *Phys. Rev. Lett.* **95**, 261602, 2005; e-Print arXiv: hep-ph/0501128.
- [15] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, “QCD and resonance physics. Theoretical foundations”, *Nucl. Phys. B* **147**, 385-447 (1979).
- [16] E. Witten, “Global Aspects Of Current Algebra,” *Nucl. Phys. B* **223**, 422 (1983).
- [17] H. U. Yee, “Holographic Chiral Magnetic Conductivity,” *JHEP* **0911**, 085 (2009) e-Print arXiv: 0908.4189 [hep-th].
- [18] H. R. Grigoryan, A. V. Radyushkin, “Anomalous Form Factor of the Neutral Pion in Extended AdS/QCD Model with Chern-Simons Term”, JLAB-THY-08-802, *Phys. Rev.* **D77**, 115024, 2008; e-Print: arXiv:0803.1143 [hep-ph]
A. S. Gorsky, A. A. Krikun, “Magnetic susceptibility of the quark condensate via holography”, ITEP-TH-04-09, *Phys. Rev.* **D79**, 086015, 2009; e-Print: arXiv:0902.1832 [hep-ph];
A. Rebhan, A. Schmitt and S. A. Stricker, “Anomalies and the chiral magnetic effect in the Sakai-Sugimoto model”, *JHEP* **1001**, 026 (2010); arXiv: 0909.4782[hep-th];
A. S. Gorsky, P. N. Kopnin, A. V. Zayakin, On the Chiral Magnetic Effect in Soft-Wall AdS/QCD, arXiv: 1003.2293[hep-ph].
- [19] A. Krikun, “Four-point correlator of vector currents and electric current susceptibility in holographic QCD,” *Phys. Lett. B* **692**, 36 (2010) e-Print arXiv:1003.1041 [hep-ph].
- [20] E. G. Thompson and D. T. Son, *Phys. Rev. D* **78**, 066007 (2008) [arXiv:0806.0367 [hep-th]].
- [21] Peter F. Kolb, Ulrich W. Heinz, SUNY-NTG-03-06, Invited review for “*Quark Gluon Plasma 3*”. Editors: R.C. Hwa and X.N. Wang, World Scientific, Singapore, 634-714 (2003).

- [22] Jens O. Andersen, Michael Strickland, Nan Su, “Three-loop HTL gluon thermodynamics at intermediate coupling”, *JHEP* **1008**, 113, 2010; e-Print arXiv: 1005.1603 [hep-ph].
- [23] K. Fukushima, D. E. Kharzeev and H. J. Warringa, “Electric-current Susceptibility and the Chiral Magnetic Effect,” Nucl. Phys. A **836**, 311 (2010); e-Print arXiv: 0912.2961 [hep-ph].
- [24] D. T. Son and M. A. Stephanov, “Axial anomaly and magnetism of nuclear and quark matter,” Phys. Rev. D **77**, 014021 (2008); e-Print arXiv: 0710.1084 [hep-ph].
- [25] A. Gorsky and M. B. Voloshin, “Remarks on Decay of Defects with Internal Degrees of Freedom,” Phys. Rev. D **82**, 086008 (2010); e-Print arXiv: 1006.5423 [hep-th].